# Select Research from Ph.D. (Brief)


## Summary

For many good reasons, Einstein's general relativity (GR) is the preferred effective theory of gravitational interactions. Yet precision cosmology may have revealed candidate discrepancies between GR and observation [1]. A lack of galactic substructure may (or may not) constitute a *small-scale crisis*, while the *Hubble tension* between early and late determinations of the Universe's expansion rate may exceed $10\%$. My Ph.D. focusses on GR and alternative *modified gravity* theories, though my future work aims to be more model independent, and closely integrated with observational cosmology.


Following my undergraduate contributions to the classical analogy between GR and Maxwell's electromagnetism [2, 3], I began my Ph.D. by addressing *where* in space a gravitational field stores its energy – an open problem in GR, and critical to gravitational wave astronomy. In 2010, a localisation scheme for weak gravitational fields was proposed in the form of the *Butcher tensor* [4]. I developed a strong-field generalisation which revealed the *Butcher tensor* to be (up to a gauge) the weak-field limit of the *Einstein pseudotensor*, a localisation devised by Einstein himself in 1916 [5]. I further proved that gravity, a *tensor* field, stores energy near a spherical mass just like a *scalar* field (fig. 1). This *Klein–Gordon correspondence* revises the gravitational binding energy of the Earth by $\sim 100$ tonnes.

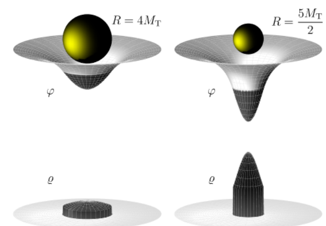


Figure 1: My gravitational 'energy', depicted here above and within a Schwarzschild (neutron) star as it collapses into a black hole.

From the second year of my Ph.D., I have been developing a new theory of gravity, which I have termed *$k$-screened gravity* [6]. In my theory, the expanding Universe cannot 'feel' its own curvature $k$, in stark contrast with GR. While consistent with e.g. Mercurial precession and the $\Lambda$CDM expansion history (fig. 2), my theory also predicts *dark radiation*, easing the Hubble tension and leading to a feature in Quanta Magazine alongside the work of Lisa Randall and Marc Kamionkowski. The $k$-screened theory is an exaple of a *torsion* theory of gravity. I was able to prove that *torsion* theories generally predict the same background cosmology as theories with scale invariance. I recently developed a mapping between such theories and the more familiar scalar-tensor theory, facilitating future investigation by the broader community. Using this mapping, I proved that cosmic torsion is very often associated with an exotic particle called a *cuscuton*, first conceived by Afshordi, and otherwise sourced from string theory and quantum gravity [7], (top arXiv papers, CMBlog). The *cuscuton* takes its name from the parasitic plant *dodder* (Latin name *Cuscuta*); it steals its behaviour from the conventional gravitons in the theory while constraining their dynamics. The *cuscuton* provides my $k$-screened gravity with its own dark energy (though not in the usual manner), even with vanishing or negative cosmological constant $\Lambda \leq 0$ (fig. 2). For weak gravity, my theory is free of unstable *ghosts* and superluminal *tachyons*, while a power counting suggests that it may be renormalisable.

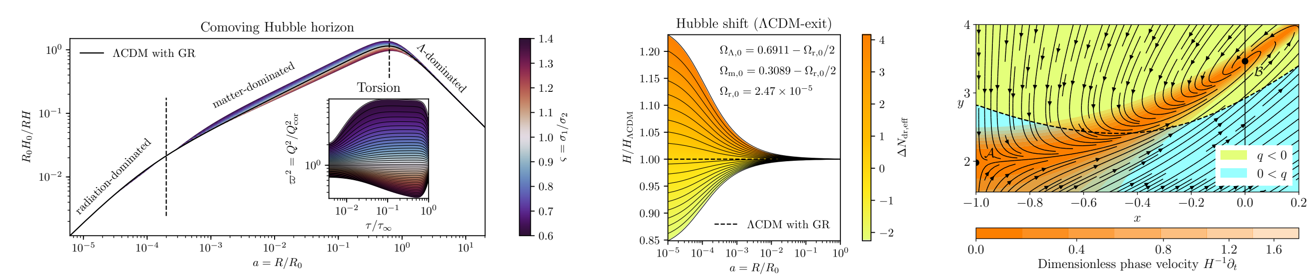


Figure 2: Left: [*Expansion rate*]$^{-1}$ vs [*Universe size*] according to my theory (colour), showing agreement is possible with GR (black line). This is remarkable, since our theories have *mathematically disconnected* Lagrangia: GR is $\mathcal{L} \sim \mathcal{R}$ whereas $k$-screened gravity is $\mathcal{L} \sim \mathcal{R}^2 + \mathcal{T}^2$. Centre: My $k$-screened gravity provides dark radiation, reheating conditions modify the subsequent early Expansion history. Right: My $k$-screened gravity provides dark energy; the Universe inevitably flows towards the accelerated expansion observed today (yellow).

# Smoking Gun for Primordial Geometry?


## Abstract

It is proposed to settle the debate over non-Riemannian spacetime geometry. For nearly 100 years, theorists have speculated that spacetime might have geometric qualities (torsion, non-metricity, scale invariance) beyond the curvature proposed by Einstein. These qualities were introduced to drive inflation across a diaspora of models, and are presumed to leave imprints on the quantum fluctuations that seed structure formation. Such imprints will be determined using a model-independent, effective field theory approach. By assuming general relativity as the low-energy, late-Universe limit, all but the least conservative models will be captured. In conjunction with bounds placed on primordial correlators by past and future cosmological surveys, the tools we develop here will orient future study in the fields of inflation and the ultraviolet completion of gravity.


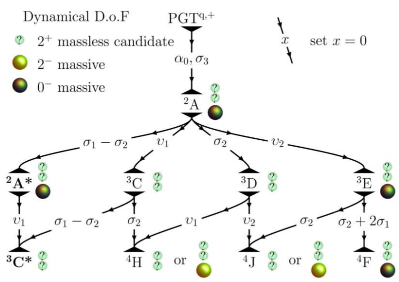


Figure 3: No shortage of inflaton candidates in non-Riemannian geometry. Here are shown massive *tordions* (torsion particles) in my $k$-screened gravity [6, 7], which is of the form $\mathcal{L}\sim\mathcal{R}^2+\mathcal{T}^2$ with $\mathcal{Q}=0$.

According to GR, matter tells spacetime how to curve, while curvature tells matter how to move. Yet the curvature $\mathcal{R}^{\alpha}{}_{\beta\mu\nu}$ is by no means the only geometric flavour we can ascribe to spacetime. Principally, one may have *torsion* $\mathcal{T}^{\alpha}{}_{\mu\nu}$ and *non-metricity* $\mathcal{Q}^{\alpha}{}_{\mu\nu}$; for our purposes it is convenient to write these tensor quantities schematically as $\mathcal{R}$, $\mathcal{T}$ and $\mathcal{Q}$. It was observed as early as 1922 that the original formulation of GR, with Lagrangian $\mathcal{L}\sim\mathcal{R}$ and assumptions $\mathcal{T}=\mathcal{Q}=0$, is not unique [8]. In a remarkable twist, there exist *teleparallel* ($\mathcal{L}\sim\mathcal{T}^2$ and $\mathcal{R}=\mathcal{Q}=0$) and *coincident* ($\mathcal{L}\sim\mathcal{Q}^2$ and $\mathcal{R}=\mathcal{T}=0$) formulations which make the same classical predictions. This *geometrical trinity* of gravity was not completed until 2019, in work which quickly attracted significant community interest [9]. I have worked extensively on these *non-Riemannian* geometries, which have proven very attractive to theorists as they naturally introduce many extra degrees of freedom (i.e. particles or fields beyond the graviton of GR) [10]. These are appealing candidates to drive inflation (fig. 3), since the gravitational coupling facilitates reheating into the primordial plasma. At a theoretical level, $\mathcal{L}\sim\mathcal{T}^2$ and $\mathcal{L}\sim\mathcal{Q}^2$ are *Yang–Mills* formulations of GR, emulating the strong and electroweak sectors of the standard model. I am interested in model-independent tests of non-Riemannian theories, for which I will develop a *geometric* GR effective field theory: (G)GREFT (fig. 4).

$$
\begin{array}{llllll}
\mathcal{L}_{\mathsf{EFT}} = & \Lambda & +{m_{\mathrm{p}}}^2\mathcal{R} & +\mathcal{R}^2+\partial^2\mathcal{R} & +\frac{1}{m^2}\mathcal{R}^3+\frac{1}{m^2}\partial^2\mathcal{R}^2+\dots & \Big\}\ \text{general relativity}+\dots \\
 & +{m_{\mathrm{p}}}^2H_0^2f(\Phi) & +{m_{\mathrm{p}}}^2f(\Phi)\mathcal{R}+{m_{\mathrm{p}}}^2f(\Phi)(\partial\Phi)^2 & +f(\Phi)\mathcal{R}^2+f(\Phi)\mathcal{R}(\partial\Phi)^2+f(\Phi)(\partial\Phi)^4 & +\frac{1}{m^2}f(\Phi)\mathcal{R}^3+\frac{1}{m^2}f(\Phi)\mathcal{R}^2(\partial\Phi)^2+\dots & \Big\}\ \dots\text{conventional inflaton OR}\dots \\
 & & \begin{array}{c}+{m_{\mathrm{p}}}^2\mathcal{T}^2+{m_{\mathrm{p}}}^2\mathcal{T}\mathcal{Q}+{m_{\mathrm{p}}}^2\mathcal{Q}^2\\+{m_{\mathrm{p}}}^2\partial\mathcal{T}+{m_{\mathrm{p}}}^2\partial\mathcal{Q}\end{array} & \begin{array}{c}+\mathcal{R}\mathcal{T}^2+\mathcal{R}\mathcal{T}\mathcal{Q}+\mathcal{R}\mathcal{Q}^2+\mathcal{R}\partial\mathcal{T}+\mathcal{R}\partial\mathcal{Q}\\+(\partial\mathcal{T})^2+\partial\mathcal{T}\partial\mathcal{Q}+(\partial\mathcal{Q})^2+\dots\end{array} & \begin{array}{c}+\frac{1}{m^2}\mathcal{R}^2\mathcal{T}^2+\frac{1}{m^2}\mathcal{R}^2\mathcal{T}\mathcal{Q}+\frac{1}{m^2}\mathcal{R}^2\mathcal{Q}^2\\+\frac{1}{m^2}\mathcal{R}^2\partial\mathcal{T}+\frac{1}{m^2}\mathcal{R}^2\partial\mathcal{Q}+\cdots\end{array} & \Big\}\ \dots\text{primordial geometry?} \\
 & \underbrace{\qquad}_{\text{late dark energy}} & \underbrace{\qquad\qquad}_{\text{early inflation}} & \underbrace{\qquad\qquad}_{\text{one-loop correction}} & \underbrace{\qquad\qquad}_{\text{two-loop correction}} &
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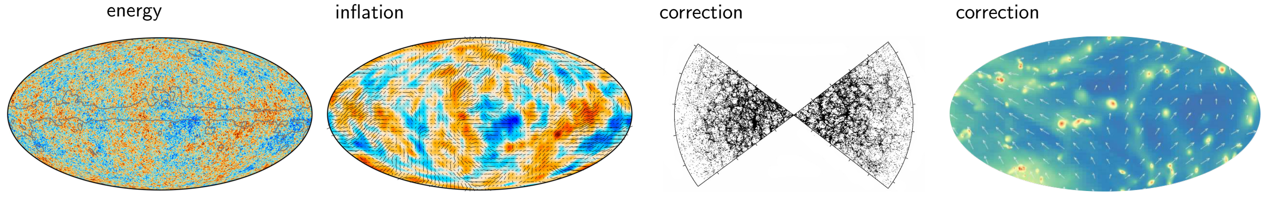

Figure 4: I am interested in finding observational signatures of early-Universe non-Riemannian geometry; a rich new field. The EFT picture makes models *predictive* at the level of the LSS and CMB statistics; a simple feasibility calculation indicates that new Wilson coefficients would grow slowly enough in (G)GREFT to maintain predictive power. Inflation could be driven by leading or loop geometric terms. I propose a multimessenger test against CMB temperature anisotropies and B-mode polarization, galaxy clustering, and (possibly) intrinsic galaxy alignment observations (illustrations adapted from Planck 2018, 2dFGRS 2001 and 2015 Horizon-AGN simulation).

The geometrical trinity suggests three promising low-energy limits from which to attempt an EFT expansion. A naïve ultraviolet embedding of the conventional $\mathcal{L} \sim \mathcal{R}$ limit is illustrated in fig. 4. In the spirit of the literature, inflation should be driven by the geometry, rather than an ad-hoc inflaton field. The low-energy limit of the theory could be 'shifted' to include a suitable inflationary mechanism which is dynamically self-removing in the late Universe; this option is illustrated in fig. 4. There are likely to be many such 'shifts' available, and these should be found systematically. Generically, this is extremely difficult. The leading terms in the EFT expansion should be unitary, requiring a delicate balancing of gravitational couplings.

The unitarity and strong-coupling problems can be seriously addressed using my powerful computer algebra package, HiGGS (**H**amiltonian **G**auge **G**ravity **S**urveyor) which probes the classical stability of such theories in the strong-field regime. Machine-assisted surveys will isolate any new 'islands' of classical stability in the theory space, lying at points close to the vertices of the geometrical trinity. The software implements the *Dirac–Bergmann* and *Castellani* algorithms for `Mathematica`'s xAct suite. While still in the development stage, the preliminary `HiGGS-v1.0` has already produced science-grade results by ruling out several promising theories of the class $L \sim \mathcal{R}^2 + \mathcal{T}^2$ with $\mathcal{Q} = 0$; this first preprint is prepared for *Phys. Rev. D* (fig. 5), with several more expected as I incorporate Weyl gauge theory in the coming year.

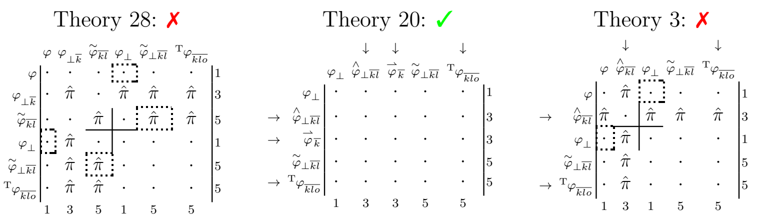


Figure 5: First-generation `HiGGS` output. Pathological *acausal* gravitons ($\hat{\pi}$) would carry information backwards through time. These calculations take days by hand; my parallelised computer algebra software performs them in minutes.

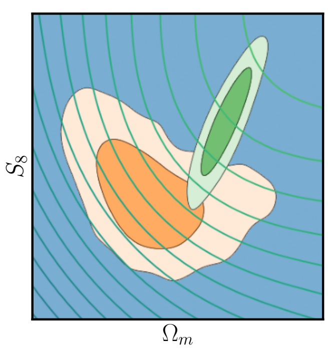


Figure 6: My method for quantifying tensions, here applied to early vs late-Universe cosmology. Cosmological surveys are DES (orange) and Planck (green), joint posteriors [*clustering parameter*] vs [*matter fraction*]. Tension shown as *discordance contours*; neural network implementation in `python`.

Apart from maintaining consistency with the $\Lambda$CDM paradigm, inflation lends predictive power to the model. A key object of study will be three-point (and higher) correlators, which translate to observable signatures. Generically, it is expected that non-Riemannian geometry will drive a variant of multi-field inflation: non-minimal coupling and time-dependent masses should lead to a rich spectrum of features in the primordial density correlators [11, 12, 7]. Comprehensive investigation will produce a dictionary from each underlying model to an expected set of *templates*. In the first instance, I will test against existing surveys such as Planck+SDSS, presumably using some variant of the modal estimator method to scan through a machine-generated template catalogue. In combination, CMB lensing and large scale structure (LSS) tracer surveys work synergistically against cosmic variance, reducing the error of the local bispectrum amplitude $f_{\mathrm{NL}}^{\mathrm{lo}}$. This is also true of future surveys (fig. 7), for which I will generate forecasts: CMB-S4 may achieve $\sigma(f_{\mathrm{NL}}^{\mathrm{lo}}) = 2.6$, but combination with upcoming photometric surveys such as Euclid or LSST clustering may even push $\sigma(f_{\mathrm{NL}}^{\mathrm{lo}}) = 0.4$: if realised, such resolving power could decide the fate of my inflationary theories [13]. LSST and Euclid will be especially important in measuring galaxy shapes; non-Riemannian theories involve high-spin fields, plausibly resulting in anisotropic primordial bispectra which translate to intrinsic galaxy alignment at low $z$ [14].

Upcoming CMB polarimetry experiments such as BICEP Array and LiteBIRD should provide orthogonal constraints on primordial tensor correlators; the non-canonical kinetic structures identified during my Ph.D. can result in blue-tilted spectra, while high-spin fields can generate TB and EB cross-spectra [15]. Finally, the EFT and Hamiltonian formulations lay a solid foundation for the BRST analysis of related gravity theories at arbitrarily high-energies.

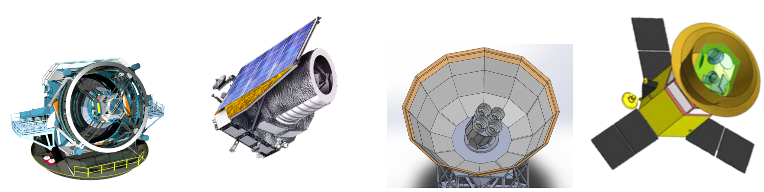

Figure 7: The next generation of deep photometric, spectroscopic (LSST, Euclid, left) and microwave (BICEP Array and LiteBIRD, right) surveys will provide stringent constraints on primordial geometry.

I will also pursue my long-standing interest in cosmological tensions. There is no 'perfect' method to determine the curvilinear direction of maximum tension between datasets, which is of critical importance when constructing salient new theories. By exporting my expertise in Hamiltonian gravity into Bayesian astrostatistics, I was able to show that the problem encodes a non-local, 6D field theory, admitting a unique solution (fig. 6). This method is distinct from leading alternatives such as curvilinear component analysis and Sammon's mapping. I will continue my contributions to this project in future years in collaboration with my current institution, with the aim of developing software for use by data scientists in disparate fields.

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